RESULTS OF INVESTIGATION OF THE SPRING SHANK DISC HARROW PERFORMANCE

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In this paper we determined the relationships between stress, stiffness coefficients, and relative and absolute deformation at each point of the spring shank of a disc harrow having a shape of an Archimedean spiral, and the parameters of the shank's geometric shape (separation distance, spiral displacement along the radial coordinate, thickness of a spring shank). Patterns of the degree of asymptotic stability (oscillation angle) of the system of working bodies of the disc harrow with spring shanks having the shape of an Archimedean spiral with different stiffness coefficients depending on the disc harrow design and technological parameters (separation distance of a spiral of the first and second row spring shanks, distance between spring shanks, diameter, attack and inclination angles of a disc working body, speed of movement) were determined. Further development of the relationship between the area of the contact zone of soil with the surface of a disc working body and the soil surface line in contact with the working body and its design and technological parameters (radius of the spherical surface, disc diameter, attack and inclination angles and working depth), that in combination with analytical parameters of stress of the visco-elastic–plastic soil will allow to determine resistive forces required for designing of spring shanks.

Keywords: disc harrow, spring shanks, soil, stress, deformation, resistive force.

1. Introduction

One of the main tasks of tillage is to create favorable environment for accumulation of nutrients and, in particular, moisture for normal development of crops. According to agronomists and soil scientists, the most important factor in such processes is a homogeneous soil structure throughout the depth of its preparation [1]. Therefore, it is necessary to improve agricultural machinery and implements in order to optimize their operation while reducing energy costs for the process [2].

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Of particular importance is the solution of these problems for tillage machines with disc working bodies, as they provide a significant share of the primary and main tillage in modern technologies of crop production [3].

A promising way to improve the quality of tillage while reducing its energy consumption is to use disc agricultural implements with working bodies on individually mounted spring shanks. This causes their oscillations due to uneven distribution of resistive forces in the soil and soil destruction at lower energy consumption and better adaptation to the field surface topography to ensure the best quality of tillage [4].

Therefore, research aimed at improving the quality and energy performance of technological processes of tillage with disc working bodies on spring shanks is relevant.

The aim of this work was to improve the efficiency of disc working bodies by substantiating the design and technological parameters of the spring shank disc harrow with different stiffness coefficients.

There is a number of publications addressing the interactions of disc working bodies with soil. The most valuable contribution was made by Shevchenko [5], Kushnarev [6], Sokht et al. [7], Hustol and Kovbasa [8], Bakum and Yashchuk [9]. Each of these works has a unique approach to solving the scientific and technical problem of interaction of a disc working body with soil environment. For example, Shevchenko [5, 10] developed the equation of motion for a material particle moving on the concave spherical surface of a disc working body. The works of Bakum and Yashchuk [9], solve the descriptive geometry problem of determining the height of ridges above the bottom of the furrow during tillage with a disc harrow. Hutsol and Kovbasa in their monography [8, 11] provide physical equations representing the relationship between stress and rates of deformations (strain rates) of soil under the action of a disc working body. Each of the presented models is used separately from each other, which leads only to a one-sided consideration of the scientific and technical problem to solve. Therefore, the following objectives were set with the aim to supplement and generalize these models: to study the movement of the soil particle on the concave spherical surface of the working body of a disc harrow and to determine the line of contact of the soil with it; to determine the area of contact zone of the soil with the surface of the working body of a disc harrow, taking into account stress arising in the soil under action of a disc working body; and to determine the components of the corresponding resistive force.

2. Analytical studies

Based on the analysis of movement of the soil particle on the concave spherical surface of the working body of a disc harrow, taking into account the resistive force of the soil layer accumulating on the disc working body, the centrifugal force and the Coriolis force, arising as a result of its rotation, in the spherical coordinate system (ψ, χ), the following system of differential equations was developed for soil particle P sliding on the sphere (Figure 1, a):

$$\begin{cases} m_p R\left(-\sin^2\psi\ddot{\chi}-\dot{\psi}^2\right)=-m_p g\cos\psi+N, \\ m_p R\left(\ddot{\psi}-\sin\psi\cos\psi\dot{\chi}^2\right)=m_p g\sin\psi+m_p q\left(1-\left(R\dot{\psi}+R\omega\cos\gamma\sin\chi\right)/V\right)/\rho- \\ -\mu N\left(R\dot{\psi}+R\omega\cos\gamma\sin\chi\right)\div \\ \div\sqrt{\left(R\dot{\psi}+R\omega\cos\gamma\sin\chi\right)^2+\left(R\sin\psi\dot{\chi}-R\omega\sin\gamma\sin\psi+R\omega\cos\gamma\cos\psi\sin\chi\right)^2}, \\ m_p R\left(\sin\psi\ddot{\chi}+2\dot{\chi}\dot{\psi}\cos\psi\right)= \\ =m_p q\left(1-\left(R\sin\psi\dot{\chi}-R\omega\sin\gamma\sin\psi+R\omega\cos\gamma\cos\psi\sin\chi\right)/V\right)/\rho- \\ -\mu N\left(R\sin\psi\dot{\chi}-R\omega\sin\gamma\sin\psi+R\omega\cos\gamma\cos\psi\sin\chi\right)\div \\ \div\sqrt{\left(R\dot{\psi}+R\omega\cos\gamma\sin\chi\right)^2+\left(R\sin\psi\dot{\chi}-R\omega\sin\gamma\sin\psi+R\omega\cos\gamma\cos\psi\sin\chi\right)^2}, \end{cases} \quad (1)$$

where m_p is the mass of a soil particle, kg; $R=d/(2\sin\zeta)$ is the radius of the sphere of a disc working body, m; d is the diameter of a disc working body, m; ζ is the half-angle at the top of the sector of a disc working body, rad; ψ, χ are spherical coordinates, rad; g is the acceleration due to gravity, m/s^2; N is the reaction force of the disc surface, N; q is the coefficient of volume compressibility, N/m^3; ρ is the volumetric weight of the soil, kg/m^3; V is the linear speed of a disc working body, m/s; μ is the coefficient of friction of the disc during its rotation; $\omega=2V/(\mu d\cos\alpha\cos\gamma)$ is the angular velocity of the particle on the disc, rad/s; α, γ are the attack and inclination angles of a disc working body, rad.

To determine the equilibrium position of the soil particle relative to the absolute space, it is assumed that $\dot{\psi}=0$, $\dot{\chi}=0$. Taking into account the corresponding transformations into the Cartesian coordinate system $x=R\sin\psi\cos\chi$, $y=R\sin\psi\sin\chi$, $z=R\cos\psi$ using Mathematica package the system of differential equations (1) was solved and the equation of the line of contact of a disc working body with soil was developed (Figure 1b):

$$z(x,y)=0.7199+1.4522\,x+2.5746\,x^2-5.72681\,y-2.2958\,x\,y+6.7766\,y^2. \quad (2)$$

Using Mathematica package, variation of the following parameters was performed: working depth h – from 0.03 m to 0.12 m, attack angle α and inclination angle γ of the working body of a disc harrow - from 0 ° to 30 °, and the values of the area of the contact zone S of soil and working body of a disc harrow were determined using the following equation: $S=\iint_{ABCD}\sqrt{1+(\partial z/\partial x)^2+(\partial z/\partial y)^2}\,dxdy$, where ABCD is a figure bounded by the lines of equation (2) and a circle of diameter d with the center at the following point: $x_c = R\sin \frac{1}{2}(\psi_{max}+\psi_{min})\cos \frac{1}{2}(\chi_{max}+\chi_{min})$, $y_c = R\sin \frac{1}{2}(\psi_{max}+\psi_{min})\sin \frac{1}{2}(\chi_{max}+\chi_{min})$, $z_c = R\cos \frac{1}{2}(\psi_{max}+\psi_{min})$.

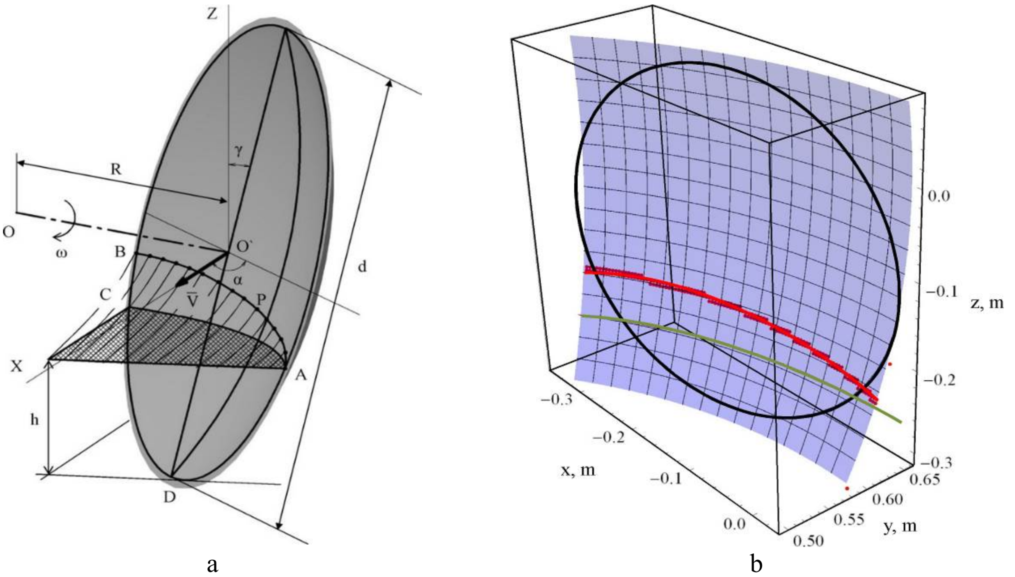


Fig. 1. Analytical model of the working body of a disc harrow (a) and graphical representation of the line of contact with soil (b)

Approximating the obtained data, we obtained a second-order regression equation for the area of the contact zone:

$$S(h,\alpha,\gamma) = -0.001857 + 0.2246\,h + 0.5729\,h^2 + 0.004337\,\alpha + 0.1081\,h\,\alpha + \\ + 0.01116\,\alpha^2 + 0.001439\,\gamma + 0.018\,h\,\gamma + 0.001978\,\alpha\,\gamma - 0.0002979\,\gamma^2. \quad (3)$$

To determine projections of resistive force of soil under the action of a disc working body upon it, by the following equations: $F_x = \iint\limits_{ABCD} \sigma_x dydz$, $F_y = \iint\limits_{ABCD} \sigma_y dxdz$, $F_z = \iint\limits_{ABCD} \sigma_z dxdy$, Hutsol and Kovbasa's study results were used [8, 11], namely, the analytical dependencies of the components of normal stress for visco-elastic-plastic soil. Using Mathematica package, variation of the following parameters was performed: attack α and inclination γ angles of the working body of a disc harrow from 0 to 30°, speed of its movement V (1–4 m/s) and working depth h (0.03–0.12 m) and the respective projections of resistive force were determined. Approximating the obtained data, regression equations were obtained for three resistive force projections (Figure 2):

$$F_x = 5627.9\,V\,(-0.0032 + h^2 + 0.0194\,\alpha^2 + \alpha\,(0.0075 + 0.0034\,\gamma) + h\,(0.391 + \\ + 0.188\,\alpha + 0.0317\,\gamma) + 0.00251\,\gamma - 0.00052\,\gamma^2)\,(\cos\alpha + \sin\alpha\,(0{,}307\cos\gamma + 0.307\sin\gamma)), \quad (4)$$

$$F_y = 1731.6\,V\,(-0.00324 + h^2 + 0.01948\,\alpha^2 + \alpha\,(0.0075 + 0.00345\,\gamma) + h\,(0.3919 + \quad (5)$$

$$+ 0.188\,\alpha + 0.0317\,\gamma) + 0.00251\,\gamma - 0.00052\,\gamma^2)\,(\cos\alpha + \sin\alpha\,(3{,}25\cos\gamma + \sin\gamma)),$$
$$F_Z = 1731.6\,V\,(-0.003242 + h^2 + 0.0194\,\alpha^2 + \alpha\,(0.0075 + 0.00345\,\gamma) + h\,(0.3919 + 0.188\,\alpha + 0.0317\,\gamma) + 0.00251\,\gamma - 0.00052\,\gamma^2)\,(\cos\alpha + \sin\alpha\,(\cos\gamma + 3.25\sin\gamma)). \quad (6)$$

The problem of deformation of the spring shank of a disc harrow was considered taking into account the following assumptions and simplifications [12]: the spring shank is absolutely elastic, i.e., its state can be described by the equation of equilibrium, Hooke's law formulas, and the dependencies between the components of the deformation tensor and the components of the displacement vector; the deformation process occurs in two directions, so we will consider a two-dimensional coordinate system; a spring shank has a spiral shape and can be described by a function in the polar coordinate system.

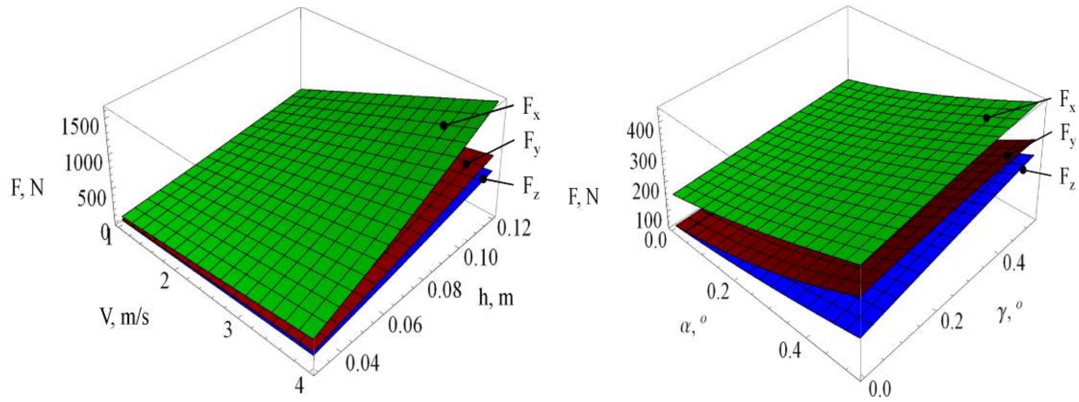
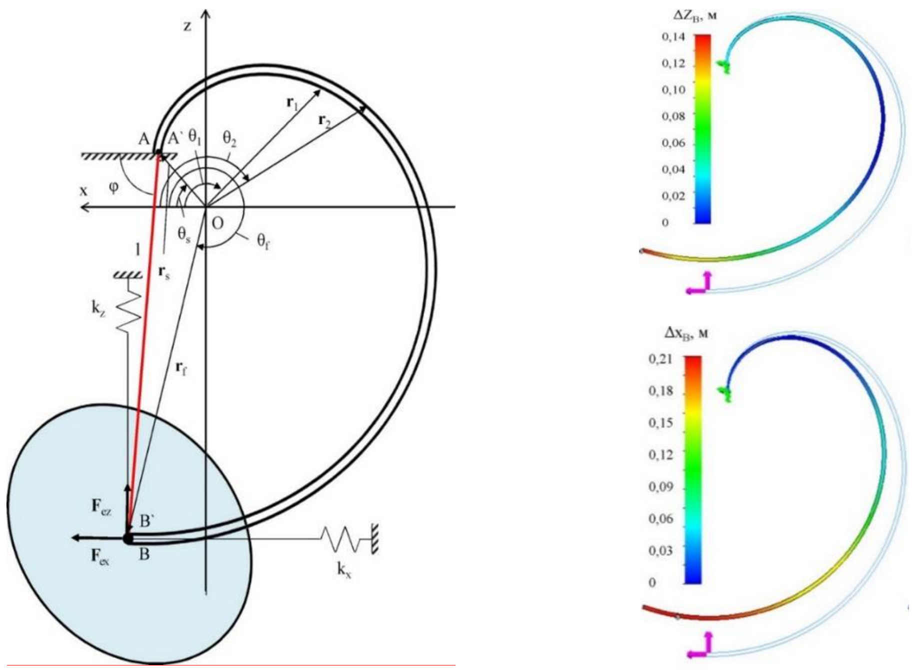


Fig. 2. Relationship between projections of resistive force and speed of movement of the working body of a disc harrow V, working depth h at α = 15°, γ = 10° (a) and attack angle α and inclination angle γ of the working body of a disc harrow at V = 3 m/s, h = 0.03 m (b)

Analytical model of the process of a spring shank deformation is shown in Fig. 3. The origin is the point O.

The function describing the boundaries of a spring shank is written in the polar coordinate system (r, θ) as: boundary A`B`: $r_1 = f_1(\theta_1)$, where $\theta_s \le \theta_1 \le \theta_f$; boundary AB: $r_2 = f_2(\theta_2)$, where $\theta_s \le \theta_2 \le \theta_f$; boundary AA`: $\theta \approx \theta_s = \text{const}$; boundary BB`: $\theta \approx \theta_f = \text{const}$, where r is the radial coordinate of a point in the polar coordinate system, m; θ is the angular coordinate of a point in the polar coordinate system, rad; subscripts 1 and 2 correspond to the inner and outer boundary of a spring shank, respectively; subscripts s and f correspond to the start and end angles of the boundary of a spring shank.



a b

Fig. 3. Analytical model of the process of deformation of a spring shank (a) and results of numerical simulation in SolidWorks Simulation software package (b)

In addition to the above description of the boundaries of a spring shank, it can also be represented as an equivalent physical-mathematical model, such as a rigid pendulum of length l, with two springs attached to its bob along Ox and Oz axes with stiffness coefficients k_x and k_z, respectively, which deflect it by the angle φ. That is, two additional elastic forces act on the bob of the pendulum along its axes Ox and Oz: F_{ex} and F_{ez}, respectively, which can be described by Hooke's law of elasticity for a spring as follows: $F_{ex} = k_x \Delta x_B$, $F_{ez} = k_z \Delta z_B$, where Δx_B and Δz_B are the absolute displacements of the point B (or B ') in the Cartesian coordinate system as a result of the deformation of a spring shank, m.

The analysis of spring shanks for disc harrows allows the first order approximation to approximate their shape to Archimedean spiral, i.e.

$$f_1(\theta) = a\theta/(2\pi) + b \text{, where } \theta_s \le \theta \le \theta_f, \quad (7)$$

$$f_2(\theta) = f_1(\theta) + h = a\theta/(2\pi) + b + h \text{, where } \theta_s \le \theta \le \theta_f, \quad (8)$$

where a is the separation distance, m; b is the displacement of the spiral along the radial coordinate, m; h is the thickness of a spring shank, m. Since the length l_k must remain constant irrespective of the separation distance a, then the displacement of the spiral along the radial distance b can be described by the following equation:

$$l_k(a) = \text{const} \Rightarrow b = l_{k0}(a_0 - a). \quad (9)$$

To determine the start θ_s and end θ_f angles, we use the conditions of horizontal and vertical tangent lines to the Archimedean spiral:

$$\frac{dx}{dz}(\theta_s) = 0, \text{ where } 0 \le \theta_s \le \frac{\pi}{2}, \text{ and } \frac{dz}{dx}(\theta_f) = 0, \text{ where } \frac{3\pi}{2} \le \theta_f \le 2\pi, \quad (10)$$

Therefore, when solving the equation (10) using Mathematica package, e.g. at a = 0.8 m, b = 0, h = 0.01 m, we obtain the shape of a spring shank described by the following equations:

$$f_1(\theta) = 0{,}8\theta/(2\pi), \qquad f_2(\theta) = 0{,}8\theta/(2\pi) + 0{,}01, \qquad 0{,}8482 \le \theta \le 4{,}9126. \quad (11)$$

Assuming the elastic properties of the shank material (steel 60C2A: modulus of elasticity E = 212000 MPa, Poisson's ratio ν = 0.28) and geometric shape parameters (11), analytical calculation performed using Mathematica package and numerical modelling using SolidWorks Simulation package [13, 14, 15] (Figure 3b), the following equations of dependency of stiffness coefficients k_x and k_z, the length l and the angle φ of an equivalent pendulum of a spring shank on the values of forces F_{ex} and F_{ez}:

$$k_x = 3457.4 + 5.16\,F_{ex} - 0.000617\,F_{ex}^2 - 3.3274\,F_{ez} + 0.000073\,F_{ex}F_{ez} + 0.00072\,F_{ez}^2, \quad (12)$$

$$k_z = 5525.6 - 4.4095\,F_{ex} + 0.000854\,F_{ex}^2 + 3.927\,F_{ez} + 0.00114\,F_{ex}F_{ez} - 0.00134\,F_{ez}^2, \quad (13)$$

$$l = 0.681 + 0.00011\,F_{ex} - 1.4\cdot10^{-8}\,F_{ex}^2 + 0.000152\,F_{ez} - 3.1\cdot10^{-8}\,F_{ex}F_{ez} + 3.4\cdot10^{-11}\,F_{ez}^2, \quad (14)$$

$$\varphi = 1.47 - 0.000129\,F_{ex} + 1.8\cdot10^{-8}\,F_{ex}^2 - 0.00011\,F_{ez} + 3.8\cdot10^{-8}\,F_{ex}F_{ez} + 1.71\cdot10^{-8}\,F_{ez}^2. \quad (15)$$

The next stage of theoretical study was focused on building mathematical models of stability of operation of the mechanical system of the disc harrow, which is a rigid frame with two rows of spherical discs mounted on separate spring shanks, and a roller [16]. At this, the discs of the first row are mounted to the disc harrow frame with spring shanks of greater stiffness compared with the spring shanks of the second row of discs. The equivalent diagram of the disc harrow (Figure 4) shows the positions of the center of mass and the point of application of the equivalent reduced resistive forces of soil acting on the working bodies of the disc harrow.

Next, we chose a fixed Cartesian coordinate system xOz. To simplify the transformations when building the calculation model, we take an additional moving Cartesian coordinate system x\`Oz\`, with its center located at the point of linkage of the disc harrow with the tractor and the horizontal axis aligned with the machine frame. We will analyze the given equivalent forces through their projections on the axes of the moving coordinate system.

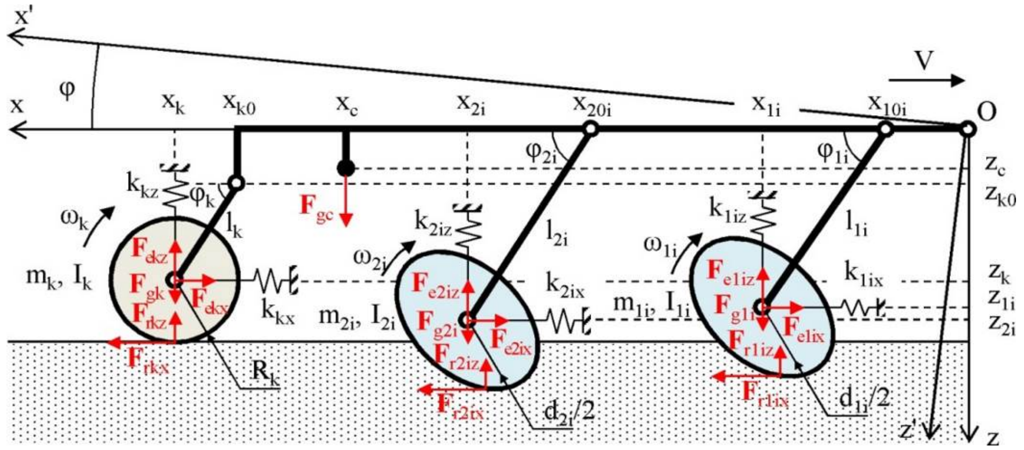


Fig. 4. Equivalent scheme of the disc harrow

Assuming that the tractor with the mounted the disc harrow performs a linear, uniform, horizontal movement, then $V_x = V = const$ and $V_z = 0$. Under the above assumptions, the system has 2 + N_1 + N_2 degrees of freedom, and, accordingly, 2 + N_1 + N_2 generalized coordinates –angles of displacement of the first row φ_{11},…, $\varphi_{1N_1} = 0$ and angles of displacement of the second row φ_{21},…, $\varphi_{2N_1} = 0$ relative to the longitudinal axis of mounting a spring shank to the frame, angle of displacement of the roller φ_k relative to the axis of mounting to the support frame, angle of rotation of the frame φ around to the longitudinal axis of mounting of the disc harrow to the tractor [17, 18]. Then the differential Lagrange equations of the second kind for the system under consideration have the following form:

$$\begin{cases} \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = Q, & \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\varphi}_{11}}\right) - \frac{\partial L}{\partial \dot{\varphi}_{11}} = Q, \\ \dots\dots\dots\dots\dots, & \dots\dots\dots\dots\dots, \\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\varphi}_{1N_1}}\right) - \frac{\partial L}{\partial \dot{\varphi}_{1N_1}} = Q, & \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\varphi}_{21}}\right) - \frac{\partial L}{\partial \dot{\varphi}_{21}} = Q, \\ \dots\dots\dots\dots\dots, & \dots\dots\dots\dots\dots, \\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\varphi}_{2N_2}}\right) - \frac{\partial L}{\partial \dot{\varphi}_{2N_2}} = Q, & \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\varphi}_{k}}\right) - \frac{\partial L}{\partial \dot{\varphi}_{k}} = Q. \end{cases} \quad (16)$$

where $L = T - U$ is the Lagrangian function of a dynamic system, J [19]: T is the kinetic energy of a system of solids, J; U is the potential energy of the system of solids, J; Q is the generalized non-conservative force (moment of force), N · m; φ is the generalized coordinate (angle of rotation), rad; $\dot{\varphi}$ is the rate of change in the generalized coordinate (angular velocity), rad/s; t is the time, s; subscripts 1 and 2 correspond to the first and second row of disc working bodies; N_1, N_2 are numbers of discs in the first and the second rows of disc working bodies; subscript k corresponds to the roller.

The kinetic energy of the system of working bodies of the disc harrow is represented by the equation:

$$
\begin{aligned}
T = {} & \frac{1}{2}M_c\Big(\big(-x_c\dot{\varphi}\sin\varphi - z_c\dot{\varphi}\cos\varphi - V\big)^2 + \big(x_c\dot{\varphi}\cos\varphi - z_c\dot{\varphi}\sin\varphi\big)^2\Big) + \\
& + \sum_{i=1}^{N_1}\Bigg(\frac{\pi}{3}\rho_{st}\Big(R_{1i}^3 - \big(R_{1i}-\delta_{1i}\big)^3\Big)\Bigg(1-\sqrt{1-\frac{d_{1i}^2}{4R_{1i}^2}}\Bigg)\times \\
& \times\Big(\big(-l_{1i}\dot{\varphi}_{1i}\sin\varphi_{1i}\cos\varphi - \big(l_{1i}\cos\varphi_{1i} + x_{10i}\big)\dot{\varphi}\sin\varphi - l_{1i}\dot{\varphi}_{1i}\cos\varphi_{1i}\sin\varphi - l_{1i}\dot{\varphi}\sin\varphi_{1i}\cos\varphi - V\big)^2 + \\
& + \big(-l_{1i}\dot{\varphi}_{1i}\sin\varphi_{1i}\sin\varphi + \big(l_{1i}\cos\varphi_{1i} + x_{10i}\big)\dot{\varphi}\cos\varphi + l_{1i}\dot{\varphi}_{1i}\cos\varphi_{1i}\cos\varphi - l_{1i}\dot{\varphi}\sin\varphi_{1i}\sin\varphi\big)^2\Big) + \\
& + \frac{\pi}{5}\rho_{st}\Big(R_{1i}^5 - \big(R_{1i}-\delta_{1i}\big)^5\Big)\Bigg(1-\sqrt{1-\frac{d_{1i}^2}{4R_{1i}^2}}\Bigg)\omega_{1i}^2\Bigg) + \sum_{i=1}^{N_2}\Bigg(\frac{\pi}{3}\rho_{st}\Big(R_{2i}^3 - \big(R_{2i}-\delta_{2i}\big)^3\Big)\Bigg(1-\sqrt{1-\frac{d_{2i}^2}{4R_{2i}^2}}\Bigg)\times \\
& \times\Big(\big(-l_{2i}\dot{\varphi}_{2i}\sin\varphi_{2i}\cos\varphi - \big(l_{2i}\cos\varphi_{1i} + x_{20i}\big)\dot{\varphi}\sin\varphi - l_{2i}\dot{\varphi}_{2i}\cos\varphi_{2i}\sin\varphi - l_{2i}\dot{\varphi}\sin\varphi_{2i}\cos\varphi - V\big)^2 + \\
& + \big(-l_{2i}\dot{\varphi}_{2i}\sin\varphi_{2i}\sin\varphi + \big(l_{2i}\cos\varphi_{2i} + x_{20i}\big)\dot{\varphi}\cos\varphi + l_{2i}\dot{\varphi}_{2i}\cos\varphi_{2i}\cos\varphi - l_{2i}\dot{\varphi}\sin\varphi_{2i}\sin\varphi\big)^2\Big) + \\
& + \frac{\pi}{5}\rho_{st}\Big(R_{2i}^5 - \big(R_{2i}-\delta_{2i}\big)^5\Big)\Bigg(1-\sqrt{1-\frac{d_{2i}^2}{4R_{2i}^2}}\Bigg)\omega_{2i}^2\Bigg) + \frac{1}{2}\pi L_k\rho_{st}\Big(R_k^2 - \big(R_k-\delta_k\big)^2\Big)\times \\
& \times\Big(\big(-l_k\dot{\varphi}_k\sin\varphi_k\cos\varphi - \big(l_k\cos\varphi_k + x_{k0}\big)\dot{\varphi}\sin\varphi - l_k\dot{\varphi}_k\cos\varphi_k\sin\varphi - \big(l_k\sin\varphi_k + z_{k0}\big)\dot{\varphi}\cos\varphi - V\big)^2 + \\
& + \big(-l_k\dot{\varphi}_k\sin\varphi_k\sin\varphi + \big(l_k\cos\varphi_2 + x_{k0}\big)\dot{\varphi}\cos\varphi + l_k\dot{\varphi}_k\cos\varphi_k\cos\varphi - \big(l_k\sin\varphi_k + z_{k0}\big)\dot{\varphi}\sin\varphi\big)^2\Big) + \\
& + \frac{\pi L_k}{4}\rho_{st}\Big(R_k^4 - \big(R_k-\delta_k\big)^4\Big)\omega_k^2
\end{aligned}
\tag{17}
$$

where M_c is the mass of the disc harrow frame, kg; x_c, z_c are the coordinates of the center of mass of the disc harrow in projections on the axis of the coordinate system xOz, m; V is the speed of movement of the machine, m/s; ρ_{st} is the density of the material of a disc working body, kg/m^3; R is the radius of the sphere of a disc working body, m; δ is the thickness of a disc working body, m; d is the diameter of a disc working body, m; l is the distance from the joint of the shank and the center of mass of the working body of a disc harrow, m; ω is the angular (rotational) velocity of the working body, rad/s; x_{10}, z_{10}, x_{20}, z_{20} are the coordinates of the joint of the first and second row spring shanks with the frame of the disc harrow, m; x_{k0}, z_{k0} are the coordinates of the joint of the roller rack with the frame of the disc harrow, m; R_k is the outer radius of the roller of the disc harrow, m; δ_k is the wall thickness of the roller of the disc harrow, m; L_k is the length of the disc harrow roller, m.

The potential energy of the system of working bodies of the disc harrow is represented by the equation:

$$U = M_c g(x_c \sin\varphi + z_c \cos\varphi) + \sum_{i=1}^{N_1} m_{1i} g((l_{1i}\cos\varphi_{1i} + x_{10i})\sin\varphi + l_{1i}\sin\varphi_{1i}\cos\varphi) +$$
$$+ \sum_{i=1}^{N_1} \frac{k_{1ix}}{2} l_{1i}^{\,2}(\cos\varphi_{1i} - \cos(\varphi_{1i} - \varphi))^2 + \sum_{i=1}^{N_1} \frac{k_{1iz}}{2} l_{1i}^{\,2}(\sin\varphi_{1i} - \sin(\varphi_{1i} - \varphi))^2 +$$
$$+ \sum_{i=1}^{N_2} m_{2i} g((l_{2i}\cos\varphi_{2i} + x_{20i})\cos\varphi_{2i}\sin\varphi + l_{2i}\sin\varphi_{2i}\cos\varphi) + \sum_{i=1}^{N_2} \frac{k_{2ix}}{2} l_{2i}^{\,2}(\cos\varphi_{2i} - \cos(\varphi_{2i} - \varphi))^2 + \quad (18)$$
$$+ \sum_{i=1}^{N_2} \frac{k_{2iz}}{2} l_{2i}^{\,2}(\sin\varphi_{2i} - \sin(\varphi_{2i} - \varphi))^2 + m_k g((l_k\cos\varphi_2 + x_{k0})\cos\varphi_k\sin\varphi + (l_k\sin\varphi_k + z_{k0})\cos\varphi) +$$
$$+ \frac{k_{kx}}{2} l_k^{\,2}(\cos\varphi_k - \cos(\varphi_k - \varphi))^2 + \frac{k_{kz}}{2} l_k^{\,2}(\cos\varphi_k - \cos(\varphi_k - \varphi))^2.$$

The equation for generalized non-potential forces (moments of forces) of the system of solids is:

$$Q = \sum_{i=1}^{N_1} F_{r1ix}((l_{1i}\cos\varphi_{1i} + x_{10i})\cos\varphi - l_{1i}\sin\varphi_{1i}\sin\varphi - Vt) +$$
$$+ \sum_{i=1}^{N_1} F_{r1iz}((l_{1i}\cos\varphi_{1i} + x_{10i})\sin\varphi + l_{1i}\sin\varphi_{1i}\cos\varphi) +$$
$$+ \sum_{i=1}^{N_2} F_{r2ix}((l_{2i}\cos\varphi_{2i} + x_{20i})\cos\varphi - l_{2i}\sin\varphi_{2i}\sin\varphi - Vt) +$$
$$+ \sum_{i=1}^{N_2} F_{r2iz}((l_{2i}\cos\varphi_{2i} + x_{20i})\cos\varphi_{2i}\sin\varphi + l_{2i}\sin\varphi_{2i}\cos\varphi) + \quad (19)$$
$$+ 0{,}86\iota\sqrt{(\pi L_k \rho_{st}(R_k^2 - (R_k - \delta_k)^2)g)^4 / (qL(2R_k)^2)} \times$$
$$\times\left(\left(f_k + \frac{\eta_k}{R_k}\right)((l_k\cos\varphi_k + x_{k0})\cos\varphi - (l_k\sin\varphi_k + z_{k0})\sin\varphi - Vt) +\right.$$
$$\left. + ((l_k\cos\varphi_2 + x_{k0})\cos\varphi_k\sin\varphi + (l_k\sin\varphi_k + z_{k0})\cos\varphi)\right)$$

where F_{rix}, F_{riz} are the forces of soil resistance to the disc working body along the axes Ox i Oz, respectively, N; f_k is the coefficient of sliding friction of the disc of the disc harrow; η_k is the coefficient of rolling friction of the roller of the disc harrow; ι is the coefficient, which takes into account the additional resistance to soil deformation caused by non-smooth elements of the roller, N/m; q is the coefficient of volume deformation of the soil, N/m^3.

Solution of the system of simultaneous equations (16)–(19), (4)–(6) i (12)–(15) was carried out using Mathematica package. Assuming $g = 9{,}8$ m/s^2, $q = 1{,}3\cdot10^3$ N/m^3; $\rho_{st} = 1340$ kg/m^3; $R = 0.66$ m; $h = 0{,}1$ m, the maximum oscillation amplitude of the angle of the disc harrow frame depending on its parameters in the form of a regression equation, was determined:

$$\begin{aligned}\varphi = 41{,}011+0.8437a_I - 0.96348\ a_I^2 - 26{,}26\ a_{II}+1{,}38\cdot10^{-15}\ a_I a_{II}+14{,}16\ a_{II}^2 - \\ - 3{,}24\ d + 4{,}91\ d^2 + 1{,}11\ V - 0.0782\ a_I\ V - 0.3042\ a_{II}\ V - 0.0968\ V^2 + \\ + 0.0126\ \alpha - 0.0098\ \gamma + 0.0629\ d\ \gamma + 0.00017\ \alpha\ \gamma - 75{,}25\ \Delta x - 0.89\ a_I\Delta x + \\ + 2{,}025\ a_{II}\ \Delta x - 0.0786\ V\ \Delta x + 48{,}16\ \Delta x^2.\end{aligned} \quad (20)$$

where a_I and a_{II} are the separation distances of a spiral of the first and second row spring shanks, respectively, m; Δx is the distance between the first and second row spring shanks of the disc harrow, m; d is the diameter of a disc working body, m; V is the speed of movement of the disc harrow, m; α is the attack angle of a disc working body of a disc harrow, m; γ is the inclination angle of a disc working body of a disc harrow, m.

3. Materials and methods of research

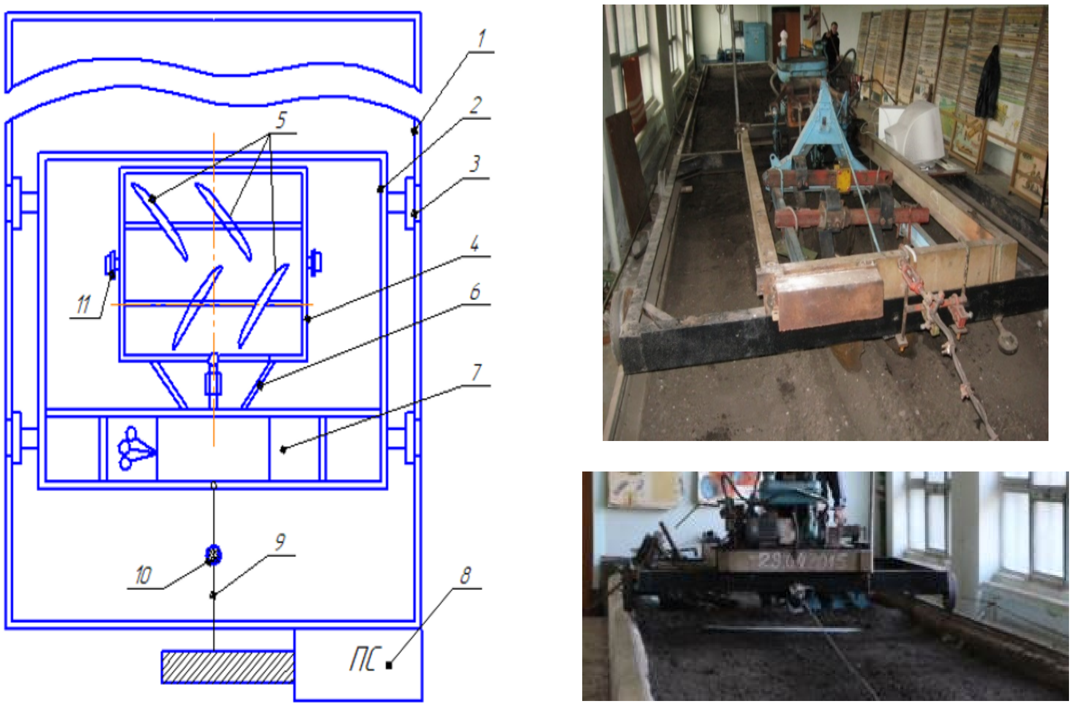
The experimental research program includes the following: the development of a laboratory method of investigation of the process of interaction of a disc working body on a spring shank with soil; the selection of the necessary standard equipment and adjustment of the test installation on a soil channel; the development of methods of experimental investigation of the performance of the spring shank disc harrow in the field; making an experimental prototype of the spring shank disc harrow; conducting experiments in accordance with the experimental design; conducting comparative field experiments.

An installation mounted on a soil channel was used for laboratory tests (Figure 5). To determine the influence of design and technological parameters of working bodies of the disc harrow on the quality of surface tillage after machine passage, three types of discs were used of the following diameters: 0.4 m, 0.5 m and 0.6 m. In addition, varying inclination angle γ (10°, 20°, 30°) and attack angle α (10°, 20°, 30°) were tested, and also the parameter of a spring shank – a separation distance a (0.6 m, 0.8 m, 1,0 m). Based on the equations (7)–(9) the shank equation can be represented as:

$$r(\theta) = a\theta/(2\pi) + 0{,}7(0{,}8 - a). \quad (21)$$

The disc harrow performance was evaluated by the following quality indicators: K_{str} – the coefficient of soil structure; R_x – the average resistive force of working bodies, kN; and $\varphi°$ – the angle of displacement of working bodies.

A complete picture of the ratio of soil aggregates of different fractions having different agronomic value can be provided by the coefficient of soil structure, which was determined by the ratio of agronomically valuable aggregates to the sum of fractions of macro- and micro-aggregates, as represented by the equation:



a b

a - schematic diagram; b - general view; 1 - guide rails; 2 - main frame; 3 - rollers; 4 - mounted movable frame; 5 - disc working bodies; 6 - linkage system; 7 - control system; 8 - drive station; 9 - rope; 10 - strain gauge; 11 - support wheel

FIG. 5. Test stand on the soil channel with equipment for testing of working bodies of the disc harrow

$$K_{str} = K_{0,25-10} / \left(K_{<0,25} + K_{>10} \right), \qquad (22)$$

where $K_{0,25-10}$ is the percentage composition of agronomically valuable fractions in the soil sample, %; $K_{<0,25}, K_{>10}$ is the percentage composition of soil fractions that are, smaller than 0.25 mm and larger than 10 mm, respectively, in the soil sample, %.

The average resistive force of working bodies was measured using a strain gauge connected to an analog-to-digital converter with subsequent computer processing of the readings. Angle of displacement of working bodies was measured using an accelerometer (acceleration sensor MPU6050), which was installed on the mounted system.

The process of experimental investigation of surface tillage can be narrowed down using a four-factor second-order D-optimal Box–Behnken design (27 experiments), which will allow to build a mathematical model that describes with sufficient accuracy the influence of factors on the technological process [20]. Speed of movement of a working body was fixed, 1 m/s. Working depth was set at 0.1 m. To ensure equal working depth of working bodies, after the control experiment, the position of the discs was not changed. The radius of the disc sphere was 0.66 m. The number of disc working bodies in a row was 2. Number of rows was 2. Experimental conditions included operation of disc working bodies on one section of the soil channel with identical soil properties, at the same depth of cultivation and with the same speeds.

For validation of the theoretical provisions developed using a mathematical model, field experiments were carried out using disc agricultural implements manufactured by Lozova Machinery, LLC (TOV). On the basis of previous theoretical studies and laboratory tests, an experimental prototype of the disc harrow shanks was developed with working bodies on individually mounted spring. (Figure 6).

According to previous theoretical studies and laboratory tests, a disc working body of diameter d = 0.56 m was used. Radius of the disc sphere was 0.66 m. Number of disc working bodies = 2 per row. Number of rows = 2. Inclination angle of a disc working body γ = 22 °, attack angle α = 31 °. Study factors included: factor A – positioning of spring shanks in both rows (a_I = 0.8 m, a_{II} = 0.6 m; a_I = 0.8 m, a_{II} = 0.8 m; a_I = 0.6 m, a_{II} = 0.8 m); factor B – distance between the two rows of disc working bodies Δx (0.6 m, 0.9 m); factor C – speed of the machine movement V (1 m/s, 2,5 m/s, 4 m/s).

a

b

Fig. 6. General view (a) and working bodies of the experimental prototype (b) of the disc harrow DL-5 with the tractor XTZ-17022

Disc agricultural implement performance was evaluated by the following quality indicators: K_{str} – the coefficient of soil structure; R_x – the average resistive force of the machine, kN; $\varphi°$ – the angle of displacement of working bodies.

The system for measuring dynamics and energy of aggregates was used for experiments. The traction force was measured using CZLAS-4 strain gauge connected to the computing module.

4. Research results

Based on the results of laboratory tests of interactions of a disc working body on a spring shank with soil and subsequent processing of the obtained data using Mathematica package, the following second-order regression equations showing the dependence of study indicators on factors were obtained:

– the average resistive force of working bodies:

$$R_x = -0.91 + 1{,}08\,a - 1{,}031\,a^2 + 10{,}11\,d - 9{,}62\,d^2 - 0.01808\,\alpha + 0.04\,d\,\alpha - 0.00329\,\gamma + \quad (23)$$

$$+0.035\,d\gamma+0.000375\,\alpha\gamma-0.0002875\,\gamma^2;$$

– the average angle of displacement of working bodies:

$$\varphi=3{,}32-0.81\,a-2{,}9\,d+3{,}41\,d^2-0.018\,\alpha+0.04\,d\alpha-0.015\,\gamma+0.035\,d\gamma+0.0004\,\alpha\gamma; \quad (24)$$

– the coefficient of soil structure:

$$K_{str}=2{,}281-0.316\,a-5{,}89d+6{,}79\,d^2-0.0402\,\alpha+0.0375\,d\alpha+0.000479\,\alpha^2-$$
$$-0.02941\,\gamma+0.04\,d\gamma+0.00035\,\alpha\gamma+0.000304\,\gamma^2. \quad (25)$$

Due to the fact that for each quality indicator of the experiment different optimal values of factors were determined, the following trade-off problem was solved:

$$R_x(d,\gamma,\alpha,a)\to\min,\quad K_{str}(d,\gamma,\alpha,a)\to\max,\quad \varphi(d,\gamma,\alpha,a)\to\min. \quad (26)$$

using the scalar ranging method by minimizing the multiplicative function taking into account the weight factor of the special criterion $\dfrac{K_{str}}{\max(K_{str})}\Big/\left(\dfrac{R_x}{\max(R_x)}\,\dfrac{\varphi_x}{\max(\varphi_x)}\right)\to\max$. Using Mathematica package, the following rational design and technological parameters of the disc harrow were obtained: d = 0.56 m, γ = 22°, α = 31°, a = 0.8 m. At these parameters, the optimization criteria were K_{str} = 0.97, R_x = 2.52 kN, φ = 2.61°.

The values of the optimization criteria obtained during experimental studies of the performance of the spring shank disc harrow in the field are shown in Table 1. Comparing all the data obtained, it is seen that the conditions $R_x\to\min,\ K_{str}\to\max,\ \varphi\to\min$ are met when spring shanks are arranged in two rows at a_I = 0.6 m, a_{II} = 0.8 m and the distance between the rows of disc working bodies Δx = 0.9 m and the speed of the machine V = 1.0 m/s.

To compare the results of theoretical, laboratory and experimental studies, appropriate graphical interpretations of the solutions of the regression equations and tabular data of oscillation amplitude of disc harrow frame corner were built. (Figs.7–8).

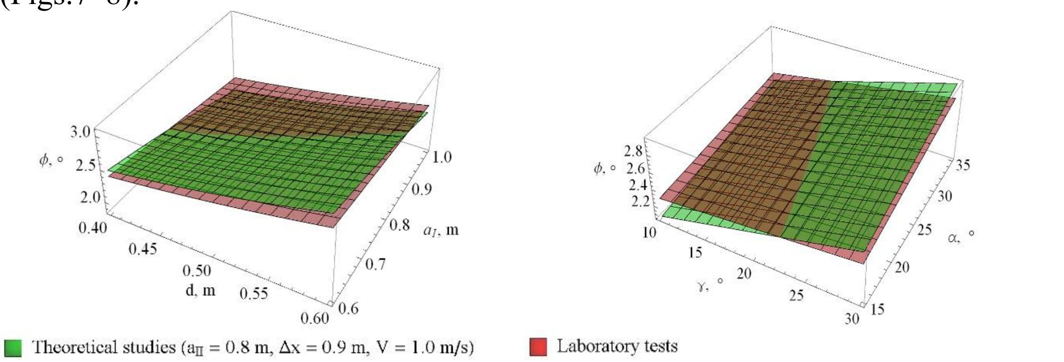
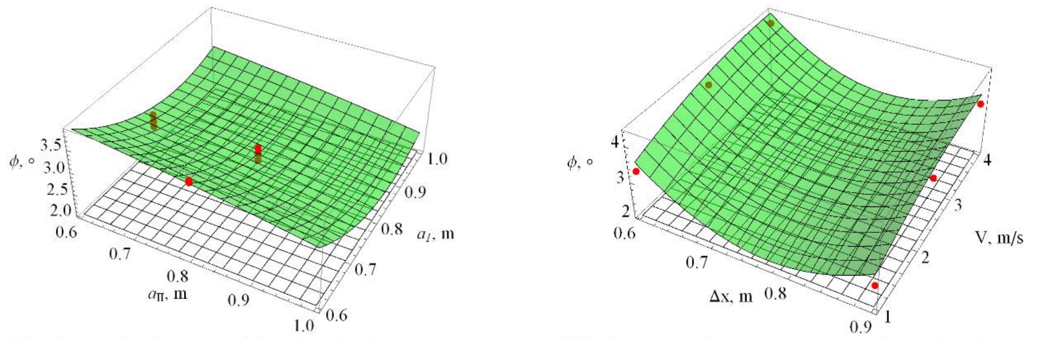


Fig. 7. Relationship between oscillation amplitude of the disc harrow frame corner and the diameter of a disc working body d, separation distance of a spiral of the spring shank of the first row a_I of working bodies and their inclination angle γ and attack angle α, as investigated during theoretical studies (18) and laboratory tests (22)



Theoretical studies (V = 1 m/s, Δx = 0.9 m, d = 0.56 m, γ = 22 °, α = 31 °)
Theoretical studies (aII = 0.8 m, aI = 0.6 m, d = 0.56 m, γ = 22°, α = 31°)
Experimental investigations

Fig. 8. Relationship between oscillation amplitude of the disc harrow frame corner and the separation distance of spirals of spring shanks of the first a_I and the second a_{II} rows of working bodies and the distance between the rows of disc working bodies Δx and the speed of machine movement V, as investigated during theoretical studies (18) experimental investigations (Table 1)

Statistical analysis comparing the results of theoretical (18), laboratory (22) and experimental (Table 1) data for functions of the oscillation amplitude of the disc harrow frame corner in the studied range of variables showed Pearson's correlation coefficient = 0.79–0.89, and Fisher's criterion F = 1.98–2.27 <Ft = 2.49, suggesting statistical adequacy of the obtained models.

Table 1

The effect of factors on the criteria of optimization of spring shank disc harrow performance in field conditions

Positioning of spring shanks in two rows (factor A)	Distance between rows of disc working bodies Δx, m (factor B)	Speed of movement of the machine V, m/s (factor C)	Average resistive force of the machine R_x, kN	Maximum angle of displacement of working bodies φ, °	Coefficient of soil structure K_{str}
a_I = 0.8 m, a_{II} = 0.6 m	0.6	1.0	27.6	4.37	0.93
		2.5	29.5	5.17	0.88
		4.0	31.6	5.46	0.64
	0.9	1.0	25.7	3.69	1,00
		2.5	27.0	4.34	0.94
		4.0	29.6	4.54	0.69
a_I = 0.8 m, a_{II} = 0.8 m	0.6	1.0	24.6	3.35	0.96
		2.5	25.3	3.93	0.90
		4.0	27.2	4.14	0.66
	0.9	1.0	23.1	2.65	1,01
		2.5	24.1	3.27	0.96
		4.0	26.0	3.43	0.71
a_I = 0.6 m, a_{II} = 0.8 m	0.6	1.0	23.4	3.28	1,05
		2.5	23.7	3.86	1,00
		4.0	24.4	4.10	0.76
	0.9	1.0	21.1	2.62	1,11
		2.5	21.7	3.27	1,07
		4.0	23.2	3.46	0.82

Positioning of spring shanks in two rows (factor A)	Distance between rows of disc working bodies Δx, m (factor B)	Speed of movement of the machine V, m/s (factor C)	Average resistive force of the machine R_x, kN	Maximum angle of displacement of working bodies φ, °	Coefficient of soil structure K_{str}
LSD_{05} (Effect of factor, %)		A	1.33 (59.31)	0.0090 (51.82)	0.0033 (12.62)
		B	1.08 (8.72)	0.0074 (22.35)	0.0027 (3.36)
		C	1.33 (14.92)	0.0090 (23.69)	0.0033 (70.54)
		AB	1.88 (0.31)	0.0128 (0.24)	0.0046 (0.02)
		AC	2.30 (1.77)	0.0157 (0.19)	0.0056 (0.02)
		BC	1.88 (0.09)	0.0128 (0.04)	0.0046 (0.01)
		ABC	3.26 (0.19)	0.0222 (0.11)	0.0080 (0.01)

LSD_{05} – Lease significant difference ($p = 0.05$)

5. Conclusions

Based on the analysis of movement of the soil particle on the concave spherical surface of the working body of a disc harrow taking into account the resistive force of the soil layer accumulating on the disc working body, the centrifugal force and the Coriolis force, arising from its rotation, a computer code was developed to determine the area and the equation of the line of contact of soil with the surface of the working body of a disc harrow depending on its design parameters (radius of the spherical surface R, disc diameter d), attack angle α, inclination angle γ and working depth h. Taking into account the obtained relationships for the area and the equation of the line of contact of soil with the surface of the working body of a disc harrow and using analytical relationships for the components of normal stress of visco-elastic-plastic soil, a computer code was developed to determine the relationship between resistive force projections and the attack angle α and inclination angle γ of the working body of a disc harrow, speed of its movement V and working depth h.

Based on the analysis of the dynamic model of deformation of a spring shank of the disc harrow of any shape, a general system of differential equations was developed, which allows to determine stress, relative and absolute deformation at each point of a spring shank. Assuming the shape of a spring shank of the disc harrow being the Archimedean spiral, i.e. the functions of its boundaries are given in the polar coordinate system $f_1(\theta) = a\theta/(2\pi) + b$, $f_2(\theta) = a\theta/(2\pi) + b + h$, where $\theta_s \le \theta \le \theta_f$, and parameters of a geometric shape a (separation distance), b (displacement of the spiral along the radial distance), h (thickness of a spring shank), its equivalent physical-mathematical model was built, in the form of a rigid pendulum of length l, with two springs attached to its bob along Ox and Oz axes with stiffness coefficients k_x and k_z, respectively, which deviate it by angle φ.

Based on the analysis, a system of differential equations was set up, describing oscillations of the frame and working bodies of the disc harrow during its movement, taking into account changes in the physical and mechanical properties of the soil. According to the developed algorithm, the regression equation was derived to determine the degree of asymptotic stability (oscillation angle φ) of the system of working bodies of the disc harrow depending on its design and technological parameters (separation distance of a spiral of the first and second row spring shanks a_I and a_{II}, distance between spring shanks Δx, diameter d, attack angle α and inclination angle γ of a disc working body, and speed of movement V).

Based on the results of laboratory tests of interaction of disc working bodies on a spring shank with soil, patterns of changes were obtained in the form of second-order regression equations for the resistive force R_x, angle of displacement of working bodies φ, coefficient of soil structure K_{str} depending on the disc diameter d, separation distance of a spring shank a, inclination angle γ and attack angle α. Due to the difference in optimal values of factors for each laboratory test indicator, a trade-off problem was solved using the scalar ranging method by minimizing the multiplicative function taking into account the weight factor of the special criterion $\frac{K_{str}}{\max(K_{str})} \Big/ \left(\frac{R_x}{\max(R_x)} \frac{\varphi_x}{\max(\varphi_x)} \right) \to \max$. Optimal rational design and technological parameters of the disc harrow were: d = 0.56 m, γ = 22 °, α = 31°, a = 0.8 m. At these parameters, optimization criteria were K_{str} = 0.97, R_x = 2.52 kN, φ = 2.61°.

As a result of experimental studies of operation of the spring shank disc harrow in the field, the pattern of changes in the resistive force of the machine R_x, angle of displacement of working bodies φ, coefficient of soil structure K_{str} depending on the position of spring shanks in the two rows, separation distances a_I, a_{II}, distance between rows of disc working bodies Δx, and speed of movement of the machine V. Comparing the results of experimental studies of operation of the spring shank disc harrow in the field, it has been established that conditions $R_x \to \min$, $K_{str} \to \max$, $\varphi \to \min$ are met for positioning of spring shanks in two rows at a_I = 0.6 m, a_{II} = 0.8 m at the distance between rows of disc working bodies Δx = 0.9 m and the speed of movement of the machine V = 1.0 m/s. At these parameters, optimization criteria were: K_{str} = 1.11, R_x = 21.1 kN, φ = 2,62°.

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